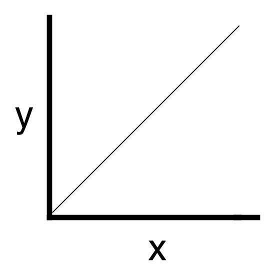
Find the Relationship: An Exercise in Graphical Analysis

In several laboratory investigations you do this year, a primary purpose will be to find the mathematical relationship between two variables. For example, you might want to know the relationship between the pressure exerted by a gas and its temperature. In another experiment, you might be asked to determine the relationship between the volume of a confined gas and the pressure it exerts. A very important method for determining mathematical relationships in laboratory science makes use of graphical methods. In this exercise, you will use a TI Graphing Calculator to help you determine several of these relationships.

EXAMPLE 1
Suppose you have these four ordered pairs, and you want to determine the relationship between x and y:

x	**y**
2	6
3	9
5	15
9	27

The first logical step is to make a graph of y versus x.



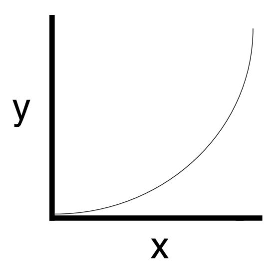
Since the shape of the plot is a straight line that passes through the origin (0,0), it is a simple *direct* relationship. An equation is written showing this relationship: y = k•x. This is done by writing the variable from the vertical axis (dependent variable) on the left side of the equation, and then equating it to a proportionality constant, k, multiplied by x, the independent variable. The constant, k, can be determined either by finding the slope of the graph or by solving your equation for k (k = y/x), and finding k for one of your ordered pairs. In this simple example, k = 6/2 = 3. If it is the correct proportionality constant, then you should get the same k value by dividing any of the y values by the corresponding x value. The equation can now be written:

y = 3•x (y varies directly with x)

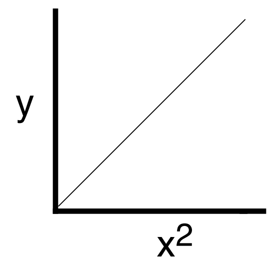
EXAMPLE 2
Consider these ordered pairs:

x	**y**
1	2
2	8
3	18
4	32

First plot y versus x. The graph looks like this:



Since this graph is not a straight line passing through the origin, you must make another graph. It appears that y increases as x increases. However, the increase is not proportional (direct). Rather, y varies *exponentially* with x. Thus y might vary with the *square* of x or the *cube* of x. The next logical plot would be y versus x^2. The graph looks like this:



Since this plot is a straight line passing through the origin, y varies with the square of x, and the equation is:

$$y = k \cdot x^2$$

Again, place y on one side of the equation and x^2 on the other, multiplying x^2 by the proportionality constant, k. Determine k by dividing y by x^2:

$$k = y/x^2 = 8/(2)^2 = 8/4 = 2$$

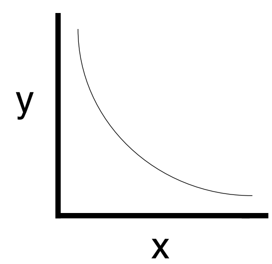
This value will be the same for any of the four ordered pairs, and yields the equation:

$$y = 2 \cdot x^2 \quad \text{(y varies directly with the square of x)}$$

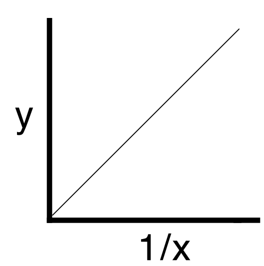
EXAMPLE 3:

x	**y**
2	24
3	16
4	12
8	6
12	4

A plot of y versus x gives a graph that looks like this:



A graph with this curve always suggests an inverse relationship. To confirm an inverse relationship, plot the reciprocal of one variable versus the other variable. In this case, y is plotted versus the reciprocal of x, or 1/x. The graph looks like this:



Since this graph yields a straight line that passes through the origin (0,0), the relationship between x and y is inverse. Using the same method we used in examples 1 and 2, the equation would be:

$$y = k(1/x) \quad \text{or} \quad y = k/x$$

To find the constant, solve for k ($k = y \cdot x$). Using any of the ordered pairs, determine k:

$$k = 2 \times 24 = 48$$

Thus the equation would be:

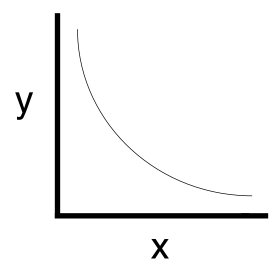
$$y = 48/x \quad \text{(y varies inversely with x)}$$

EXAMPLE 4:

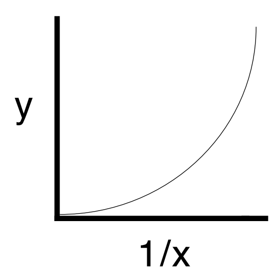
The fourth and final example has the following ordered pairs:

x	y
1.0	48.00
1.5	14.20
2.0	6.00
3.0	1.78
4.0	0.75

A plot of y versus x looks like this:

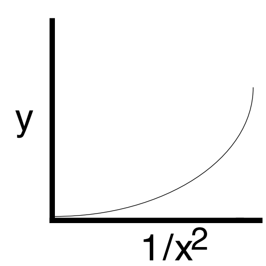


Thus the relationship must be inverse. Now plot y versus the reciprocal of x. The plot of y versus 1/x looks like this:

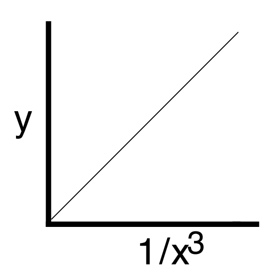


Since this graph is not a straight line, the relationship is not just inverse, but rather inverse square or inverse cube.

The next logical step is to plot y versus $1/x^2$ (inverse square). The plot of this graph is shown below. The line still is not straight, so the relationship is not inverse square.



Finally, try a plot of y versus $1/x^3$. Aha! This plot comes out to be a straight line passing through the origin.



This must be the correct relationship. The equation for the relationship is: $y = k(1/x^3)$ or $y = k/x^3$ Next, determine a value for the constant, k. For example, $k = y \cdot x^3 = (6)(2)^3 = 48$. Check to see if it is constant for other ordered pairs. The equation for this relationship is:

$y = 48/x^3$ (y varies inversely with the cube of x)

PROCEDURE

1. Pick up three problems to be worked on.

2. Turn on the TI calculator.

3. Clear any data from the first two data lists and enter the ordered pairs for one of the problems:

   TI-82 or TI-83 Calculators:

   - To view the data lists, press STAT to display the EDIT menu, and select Edit.
   - Clear any previous data from lists L_1 and L_2— move the cursor to the L_1 heading and press CLEAR, then ENTER. Do the same for the L_2 list.
   - Enter the ordered pairs of the problem into lists L_1 and L_2. First enter the "x" values into the first five lines of L_1. Press ENTER after typing in each value.
   - Enter the five "y" values into the first five rows of L_2. Press ENTER after typing each value.

4. Since you will need to use the original data pairs in Processing the Data, record the x and y values and the problem number in the Data and Calculations table.

5. Plot a graph of y vs. x:

   - Start the CHEMBIO program and proceed to the MAIN MENU.
   - Select FIT CURVE from the MAIN MENU.
   - Select LINEAR L_1, L_2. Note: The correlation coefficient, *R*, indicates how closely the linear regression curve *fits* the plotted points (that is, passes through or near the plotted points). A value of 1.00 indicates a nearly perfect fit. If your value of R isn't one, the equation isn't linear.
   - Press ENTER, then select SCALE FROM 0 . A graph of x vs. y should be displayed with a linear regression curve.

6. If the linear regression curve closely fits the plotted points, the exponent, *n*, is equal to 1. Record the value of *n* in the data table and proceed to Step 8. If the plotted points are nonlinear, decide what exponent, *n*, you want to use in the expression, x^n, in order to obtain a linear relationship. If the plot is curved, use "2" or "3" for *exponential*, "-1" for the reciprocal of x, "-2" for *inverse square*, or "-3" for *inverse cube*. Convert the x values in L_1 to x^n values in L_3:

   TI-82 or TI-83 Calculators:

   - Press ENTER. Select VIEW DATA from the MAIN MENU. To view the lists, press STAT to display the EDIT menu and then select Edit.
   - Move the cursor until the L_3 heading is highlighted. Press 2nd [L_1] ^. Type in your exponent value, *n*. Then press ENTER. Note: With negative exponents, use (–), not –.

7. Plot a graph of y vs. x^n.

   - Restart the CHEMBIO program and proceed to the MAIN MENU.
   - Select FIT CURVE from the MAIN MENU.
   - Select LINEAR L_3, L_2
   - Press ENTER, then select SCALE FROM 0. If the points on the graph are in a straight line, you have made the correct choice of the exponent, *n*. Record the value of *n* in your data table. Proceed to Step 8.
   - If the plot is still curved, repeat Steps 6-7, using a new value for the exponent.

8. Draw the graph or use the TI-Graph Link program and cable to transfer the screen image of the graph of y vs. x^n to a Macintosh or IBM computer. Print a copy of the graph. Record the problem number on the printed copy and label both axes of the paper copy of the graph.

Fill in the data table provided.

Name ______________________________ Period ______

Graphical Analysis Lab
Data Sheet

DATA AND CALCULATIONS

Problem Number ____	
X	**Y**
R = ____	k = ____

Problem Number ____	
X	**Y**
R = ____	k = ____

Problem Number ____	
X	**Y**
R = ____	k = ____

Problem Number	Equation (using x, y, & k)	Solve for "k" (find the value of k for two data pairs)	Final Equation (x, y, and value of k)	Two-point Equation
Sample	$y = k/x^2$	$k = y \bullet x^2$ $k = (4)(2)^2 = 16$ $k = (1)(4)^2 = 16$	$y = 16/x^2$	$y_1 \bullet x_1^2 = y_2 \bullet x_2^2$

1		7		13		19		25	
x	y	x	y	x	y	x	y	x	y
0.6	0.198	0.20	0.290	1.5	1.13	45	405	89	8.72
0.8	0.264	0.25	0.363	2.0	1.50	37	333	321	31.5
1.5	0.495	0.30	0.435	2.5	1.88	16	144	47	4.61
2.0	0.660	0.45	0.653	3.5	2.63	4	36	213	20.9
2.5	0.825	0.60	0.870	5.0	3.75	64	576	436	42.7
2		8		14		20		26	
x	y	x	y	x	y	x	y	x	y
32	819	0.7	16.7	2	8	4	8.0	0.1	0.002
13	135	1.2	49.0	8	128	3	4.5	0.2	0.008
43	1479	1.8	110	6	72	7	24.5	0.3	0.018
8	51.2	4.5	689	5	50	5	12.5	0.5	0.050
24	461	2.5	213	3	18	8	32.0	0.7	0.098
3		9		15		21		27	
x	y	x	y	x	y	x	y	x	y
1	2	0.4	0.192	0.1	0.0005	7	82.3	0.5	2.25
2	16	0.6	0.648	0.5	0.0625	12	415	0.8	9.22
3	54	0.2	0.024	0.3	0.0135	10	240	1.1	24.0
4	128	0.8	1.54	0.7	0.1715	15	810	1.9	123.5
5	250	0.9	2.19	1	0.5000	3	6.48	0.2	0.144
4		10		16		22		28	
x	y	x	y	x	y	x	y	x	y
2.0	1.5	2.5	10.00	6	2.50	14	5.36	5	0.16
2.5	1.2	1.5	16.67	9	1.67	25	3.00	2	0.40
1.5	2.0	12.0	2.083	10	1.50	19	3.95	8	0.10
3.0	1.0	19.0	1.316	12	1.25	36	2.08	15	0.053
5.0	0.6	5.0	5.000	20	0.75	48	1.56	0.5	1.60
5		11		17		23		29	
x	y	x	y	x	y	x	y	x	y
2.0	1.00	5.0	0.84	1.5	5.33	5	5.80	0.5	3.00
3.0	0.444	2.0	5.25	2.0	3.00	11	1.20	0.8	1.17
5.0	0.16	3.0	2.33	5.0	0.48	15	0.644	1.3	0.444
1.5	1.78	4.8	0.91	3.8	0.83	24	0.252	2.0	0.188
8.0	0.0625	7.5	0.373	6.0	0.333	30	0.161	3.0	0.0833
6		12		18		24		30	
x	y	x	y	x	y	x	y	x	y
11	0.236	0.85	1.04	0.4	78.1	4.5	0.154	2	3.00
17	0.0639	1.5	0.19	0.9	6.85	5	0.112	3	0.899
6	1.454	0.5	5.12	0.7	14.6	7	0.041	5	0.192
28	0.0143	2.0	0.08	1.2	2.89	3	0.519	7	0.070
20	0.0393	3.0	0.0237	0.6	23.2	8	0.0273	9	0.033